Project proposal for Internship EHU Quantum Center 2026

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Title: **Toward a conceptual understanding of the Osborne-Verstraete-Wootters relation for the monogamy of entanglement**

Monogamy of entanglement is a famous consequence of the restrictions that quantum mechanics places on correlations. Unlike classical correlations, quantum entanglement cannot be freely shared among many particles: if two subsystems are strongly entangled, their ability to be entangled with others is correspondingly limited.

Understanding such correlation constraints has become a central theme at the interface of quantum information theory and many-body physics. A deeper insight into these restrictions could have far-reaching implications, ranging from the design of quantum error-correcting codes to the development of improved variational ground states of condensed-matter Hamiltonians.

The OSBORNE-VERSTRAETE-WOOTTERS (OVW) inequality limits the amount of two-qubit entanglement that a given qubit A can share with any of the other qubits B_1, B_2, ..., B_{N-1} in an N-qubit state. It is one of the enigmatic results in the early theory of quantum entanglement [1]. Although the inequality has a rigorous mathematical proof [2], its physical mechanism remains only partially understood. Developing such an understanding could help to derive analogous constraints in more general settings, such as higher-dimensional systems or correlations beyond pairwise entanglement.

There exist hints that the generalized BLOCH representation of quantum states (that is, the expansion of a many-qubit density matrix in terms of the PAULI matrices, in analogy with the BLOCH sphere for a spin $\frac{1}{2}$) might play a fundamental role for understandig the OVW inequality. This is the path we will follow for this Internship Project. We will numerically investigate states of a few qubits, $4 \leq N < 10$, with the aim of identifying mathematical relations between relevant parts of the generalized BLOCH vector. Such relations may help to understand "the inner workings" of the OVW inequality.

The project, if successful, may also be continued in a longer collaboration toward an analytical proof of the identified relations and/or the OVW inequality.

Prerequisites:
- basic linear algebra of finite-dimensional quantum systems (mainly: tensor products, partial traces, matrix algebra)
- good programming skills using, e.g., Mathematica, MatLab, Python, or similar tools

Timeline: mid-June to about 7 August, 2026,
two meetings per week for the first 2–4 weeks,
afterwards once per week + additional consultation as needed

[1] V. Coffman, J. Kundu, and W.K. Wootters, Phys. Rev. A **61**, 052306 (2000).
[2] T.J. Osborne and F. Verstraete, Phys. Rev. Lett. **96**, 220503, (2006).